Railway Bridge Asset Management Modelling

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Abstract— **Railway bridges form one of the major railway asset groups with more than 35,000 bridges on the UK rail network. Additionally, the bridge structures are old with more than 50% of the population constructed over 100 years ago. Due to the unique nature of each bridge and their varied means of construction, the decision of what type of maintenance actions should be performed and when it should be carried out is a complex problem. Models can be formulated to predict the future condition of assets along with the effect that interventions such as servicing, repair, and element replacement will produce. This can be used to support this decision making process.**

This paper demonstrates a Markov modelling approach to predict the condition of individual bridge elements. For each bridge element the degradation process is determined by examining the maintenance records and analysing the times that each element takes to deteriorate to the point where maintenance of a certain severity classification is required. By combining the elemental models, an overall bridge model is formed which can be used to investigate different maintenance strategies. The model is capable of accounting for the bridge current condition, material, route criticality, structural arrangement and environment. The maintenance and renewal strategy can also be varied in the model along with the service and inspection frequency. Using the model the whole life costs can be predicted for any of the selected maintenance strategies

Keywords—**bridge, asset management, maintenance, Markov, degradation**

I. Introduction

Railway bridges are categorised into underbridges and overbridges. Underbridges carry rail traffic over obstacles, while overbridges carry another service over the railway. These bridges are made of many different materials including: stone, masonry, brick, wrought iron, steel and concrete. Metal bridges constitute a significant proportion of the UK railway bridge population and the majority of these were constructed before 1912. Many bridges have also had increased traffic loads and intensities imposed to meet the current network demands. The focus of this paper is the asset group of metal underbridges. The bridge is considered in term of its individual elements such as: decks, girders, abutments and bearings. In order to make sound decisions regarding when and how to maintain the asset the deterioration process of the asset elements over time must be well understood. Intervention costs can then be incorporated into the model allowing the total maintenance costs to be projected as well as investigate the whole life cycle costs (WLCC) of assets afforded by different maintenance strategies.

There are several models which have been developed to predict the condition of a bridge for use as a support tool for bridge asset management. These models can be classified into Markov, semi-Markov and Probabilistic models.

Jiang et al [1] and Robelin & Madanat [2] explained the use of Markov models in predicting the deterioration rate of bridges. The deterioration rate of the bridge is reflected in the state transition probability matrix which was estimated based on the condition score data of bridges at different ages. The method used in obtaining the transition probabilities minimises the absolute difference between the expected value of the condition rating from the Markov chain and the actual average condition rating from the database. Cesare et al [3], Ortiz-García et al [4] and Chase & Gaspa [5] presented real applications of Markov models to the evaluation of bridge deterioration. The studies were carried out based on the data of bridges in different states in the US. The data contains bridge element condition ratings on a scale from 1 to 7, with 7 being ‘as new’ and 1 being the worst condition. The Markov model was then applied to predict the evolution of the average condition rating of a set of bridges and the expected value of the condition rating for a single bridge. Morcous [6] investigated the effect of a non-constant inspection period on the transition matrix and proposed that the transition probabilities should be updated using Bayes’ rules for more accurate modelling.

While the Markov model is based on the assumption of an exponential distribution for duration (sojourn) times in specific bridge conditions, semi-Markov models use different distributions (often the Weibull distribution) to model these duration times.

Ng and Moses (1998) [7] discussed the use of semi-Markov processes in modeling bridge deterioration. Each state’s sojourn time distribution parameters were estimated using the difference between the two age distributions for the respective states. The study was based on real bridge condition data, however, the condition data is for the whole bridge, not bridge elements. In Kleiner [8], the author discusses modelling the waiting time of the process in any state as a random variable with a two-parameter Weibull probability distribution. The application of the model is then demonstrated based on hypothetical data, which was obtained from expert opinion and perception. Having the Weibull distribution parameters defined by the experts, the transition matrix was then obtained and the future condition of the assets was predicted. Empirical models which used real condition data can be found in Sobanjo et al [9], Mishlani and Madanat [10] and Yang et al [11]. Weibull distributions were fitted to the times of a bridge component remaining in a particular condition rating. The transition probability between states can be calculated at any point in time employing a semi-Markov model to account for the non-constant deterioration rate of the bridge element.

Probabilistic models were developed by Agrawal et al. [12], where the author studied 17,000 highway bridges in New York State with historical data available from 1981 to 2008. Again the bridge component condition ratings were on a scale from 1 to 7. The approach fitted a Weibull distribution to the durations that a bridge element stays in a particular condition then calculates the mean time of staying in that particular state. The mean duration for each different condition rating is calculated by accumulating the mean durations of the previous states. These means are then plotted on a graph of condition ratings against age and a third degree polynomial fitted to show the deterioration rate. Frangopol et al [13] took a different approach and developed a reliability index that measures the bridge safety instead of condition and the deterioration rate of a bridge is the rate of deterioration of the reliability index.

Markov, Semi-Markov and Probabilistic approaches have previously been applied to bridge assessments. There are however limitations in these models such as: their basis on condition rating data which is not an ideal for the determination of degradation processes or maintenance models, the estimation of the transition probabilities is significantly affected by prior maintenance actions (i.e. a rise in condition score) (Agrawal et al [12]); the effects of maintenance on components are not captured.

The bridge model in this paper addresses these deficiencies by the use of historical maintenance data instead of condition data. This gives the time to an event when an intervention was carried out. The model is a Markov model that represents the life of bridge components taking account of their current condition, material, structural type and environment. The model is also capable of accounting for the maintenance strategy, inspection interval, servicing interval and the repair delay time.

II. Bridge Element Condition and Intervention Types

Over time, the bridge element condition deteriorates and structural defects appear which trigger different types of intervention. Different components of the bridge are constructed from different materials and would experience different levels of degradation. The maintenance actions required by each component of the bridge would be different. There are four intervention categories considered which are given in Table 1. Servicing is the only type of maintenance which does not change the state of the component, servicing will slow down the degradation rate. Minor repair, major repair and replacement are assumed to restore the component condition to as good as new. These three interventions can be carried out when the component reaches the *good*, *poor* or *very poor* state from the *as new* condition.

III. Analysis of Bridge Element Deterioration

For each bridge element, the degradation process is determined by studying the maintenance records and analysing the times that each element takes to deteriorate to the point where a certain type of maintenance is required. From the database, for each bridge component, *i*, of the same type and material, the time to reach state *j* from new,

TABLE I
Intervention Categories and Level of Defects

Maintenance type	**Definition**			
Servicing	Activities that protect the structure from the source that drives the degradation process (eg. Waterproofing).			
Minor repair	Minor repair implies the restoration of the structure element from the good condition to the as new condition. Components in the good condition can experience the following defects			
	Metal	*Concrete*	*Timber*	*Masonry*
	Minor corrosion, tear	Spalling, small cracks, exposed of secondary reinforcement	Surface softening, splits	Spalling, pointing degradation water ingress
Major repair	Major repair implies the restoration of the structure element from the poor condition to the as new condition. Components in the poor condition can experience the following defects			
	Metal	*Concrete*	*Timber*	*Masonry*
	Major corrosion, loss of section, fracture, crack welds	Exposed of primary reinforcement	Surface and internal softening, crushing, loss of timber section	Spalling, hollowness, drumming
Replacement	Complete replacement of a component or the whole bridge. Components in the very poor condition can experience the following defects			
	Metal	*Concrete*	*Timber*	*Masonry*
	Major loss of section, buckling, permanent distortion	Permanent structural damage	Permanent structural damage	Missing masonry, permanent distortion

$T_{i,j}^{L}$, is calculated. $T_{i,j}^{L}$ is usually called the time to failure and the term 'failure' used here does not mean the physical failure of a component but indicates the time to the point when repair is necessary. It is important when analysing the lifetime data of a component to account for both complete data, $T_{i,j}^{L}$ and censored data, $T_{i,j}^{C}$. Complete data indicates the time of reaching state *j* from the new condition. Censored data is incomplete data where it has not been possible to measure the full lifetime. This may be because the component was replaced, for some reason, prior to reaching the condition *j* and so the full life has not been observed. The components life is however known to be at least $T_{i,j}^{C}$. Having obtaining these data, the transition rates, λ_j between the new state to state *j* can be estimated using Equation 1.

$$\lambda_j = \frac{\sum_{i=1}^{n} N_i}{\sum_{i=1}^{n} [T_{i,j}^{L} + T_{i,j}^{C}]} . \qquad (1)$$

where

N_i is the number of repairs on a single component

n is the number of component of type i studied

Assuming that the degradation rate is constant, the mean time to an intervention of type *j* (MTTI) can be calculated as the inverse of the degradation rate. The deterioration rates for the four different main bridge components of different materials are presented in Table 2. Considering the bridge deck, it shows that, the rate of timber deckings reaching a point where they would require a minor repair is considerably higher than the corresponding rates of metal and concrete decking. This indicates that a timber deck would need more maintenance than the metal and concrete deckings. While the average replacement rate of timber decking is after 45 years, metal decking life is about three times longer (136 years). For concrete decking, it is only needed to be replaced after 187 years, it has the longest working life, 4 times longer than timber deckings and 1.5 times more than that of metal. Note that there were not enough data to allow the replacement rate of the abutment to be estimated.

TABLE 2
DEGRADATION RATES AND MEAN TIME TO AN INTERVENTION OF BRIDGE ELEMENTS

Bridge Component	Material	Condition	Intervention	λ ($year^{-1}$)	MTTI (year)
GIRDER	Metal	Good	Minor Repair	0.03666	27.28
		Poor	Major Repair	0.01845	54.21
		Very Poor	Replacement	0.00461	216.84
DECK	Metal	Good	Minor Repair	0.01591	62.85
		Poor	Major Repair	0.01136	88.00
		Very Poor	Replacement	0.00734	136.32
	Concrete	Good	Minor Repair	0.01327	75.34
		Poor	Major Repair	0.00733	136.43
		Very Poor	Replacement	0.00533	187.51
	Timber	Good	Minor Repair	0.10885	9.19
		Poor	Major Repair	0.03173	31.52
		Very Poor	Replacement	0.02212	45.20
BEARING	Metal	Good	Minor Repair	0.02284	43.78
		Poor	Major Repair	0.01845	52.19
		Very Poor	Replacement	0.00461	67.18
ABUTMENT	Masonry	Good	Minor Repair	0.01925	51.94
		Poor	Major Repair	0.01845	100.87
		Very Poor	Replacement	-	-

IV. BRIDGE MODEL

This section contains a description of the construction of a continuous-time Markov model of the degradation, inspection, servicing and maintenance of the bridge components. The model can be used to investigate the effects of different maintenance strategies. A simulation can be carried out on each individual component and the results combined together to form a bridge model. A lifetime duration, over which the predictions will be made, has been assumed to be 60 years.

A. *Inspection and Repair Policy*

Under normal management, all bridges and their components are inspected after a certain period of time. At the point of inspection, the current state of the bridge components is identified. Bridge components can either reside in a condition that no repair is needed or have reached the condition where a certain type of repair can be carried out. If a change in the state of the element (i.e. the moving of the state from poor to very poor) happens in between two inspections, the failure is unrevealed until the second inspection. Moreover, at this point, a maintenance decision can be made to repair the component or let it continue to deteriorate to a poorer state. Figure 1 shows the Markov state diagram that was developed to model the deterioration and repair process of an element. The component starts in new condition (State 1) and deteriorates to State 2 where a minor repair can be performed. Following an inspection, if it is revealed that the component is in State 2, the element can either be scheduled for repair (State 3) or left to deteriorate (State 4) to poorer state. The option to carry out repair is achieved by setting M1=1, this will transfer the probability of a component in the good condition (State 2) to the state where the minor repair is performed (State 3). In contrast, by setting M1=0, the probability of a component in good condition can only be transferred to State 4 (no repair happens) hence the component is left to deteriorate without any intervention. A similar process applies when the component deteriorates to a state where a major repair is necessary to return it to the as new condition (State 5), the options for repair or no repair is set by M2 and represented by States 6 and 7. Note that State 8 is when the component is in a very poor condition and cannot deteriorate any further, therefore the only option is to repair (State 9) and M3 should always be 1 since the component should be repaired as soon as it reaches the very poor condition. The effects of the inspection and repair options require the model to be modelled in two phases: the first phase is the continuous phase, modeling the degradation and repair processes, between any two inspections and the second phase is at the point of inspection where the condition of a bridge element is revealed and the decision of whether to repair or not is made. There are four maintenance strategies possible in this model and are described in table 3.

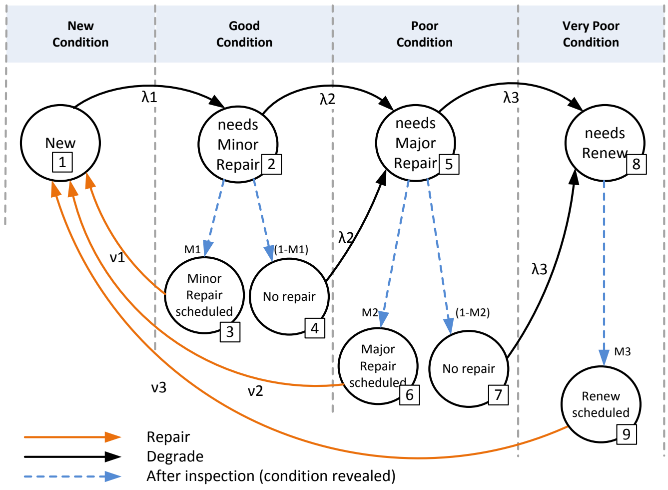

Figure 1: Markov state transition diagram modelling the service life of a component

TABLE 3
MAINTENANCE STRATEGIES AND CORRESPONDING MODEL PARAMETERS

Strategy	Model parameters			Action
Strategy 1	M1=1	M2=1	M3=1	Repair as soon as the component is identified to be in a state where repair is necessary, then it is carried out.
Strategy 2	M1=0	M2=1	M3=1	Repair when the component is identified in the state where a major repair is required i.e. repair when the component reaches poor condition.
Strategy 3	M1=0	M2=0	M3=1	Repair when the component is identified as being in the state where renewal is needed i.e. repair when the component reaches very poor condition.
Strategy 4	M1=0	M2=0	M3=0	No repair, component is allowed to deteriorate without any interventions

B. Servicing interval

A factor that changes the degradation rate is the servicing interval. Servicing work does not change the state of the component but helps slow down the deterioration process and increases the time until the next intervention is needed. A simple study was carried out to establish the effects of the service interval on the deterioration rates by comparing the degradation rates of the same group of components that have different servicing intervals recorded. The deterioration rates of a component can then be plotted against different servicing interval. Assuming a linear relationship it was found that if a metal girder is serviced every 20 years, the rate of deterioration from as new to the good condition increases by 2 times compared to one service every year. Under these circumstances the rate of it moving to a poor condition is almost 2.5 times greater. For a metal deck that is being serviced every 20 years, it also shows that the degradation rate from as new to a good condition is twice times the rate when the metal deck is being service every year, the rates from as new to poor and very poor condition also increase by 1.8 and 1.3 times respectively. The servicing effects on the deterioration rates of other components were also investigated and these effects were included in the model.

C. Transition rates

The deterioration rate of an element depends on the environment as well as the servicing frequency. A component that operates in an aggressive environment will deteriorate faster and a component serviced more frequently would have a lower deterioration rate. The deterioration rate estimated using Equation 1 should then be updated by:

$$\lambda_j' = \lambda_j \times E \qquad (2)$$

where
E is the environment adjustment factor corresponding to different environments (benign, moderate, aggressive)

The deterioration rates established above govern the process from the 'as new' state. The Markov model needs the transition rates between two adjacent states (good to poor, poor to very poor). The rate from state i to state j can be estimated as the inverse of the mean time reaching state j from state i, $MTTF_{i,j}$:

$$MTTF_{i,j}=MTTF_{j,1}-MTTF_{i,1} \qquad (3)$$

Giving:

$$\lambda_{ij} = \frac{\lambda_i \lambda_j}{\lambda_i - \lambda_j} \qquad (4)$$

The repair rates, the inverse of the mean time to repair (MTTR), ν_1, ν_2, ν_3 are also included in the model. The time to repair consists of two main components:

- the time to schedule the repair
- the time of the actual repair work being carried out.

The schedule time for the work is defined as the duration between when the work was raised and when the work actually starts. The time of repair is calculated as the duration of the work, as recorded in the historical data. Thus the repair rate can be calculated as:

$$\nu = \frac{1}{MTTR} \qquad (5)$$

As mentioned earlier, there are two phases for the model calculations:

The first phase is the continuous phase between any two inspections, the transition rate matrix (Equation 6) is based on the deterioration rates and repair rates discussed above. A numerical solution scheme is used to solve the Markov model.

$$[A] = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2 & 0 & \lambda_2 & 0 & 0 & 0 \\ \nu_1 & 0 & -(\nu_1+\lambda_2) & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_3 & 0 & \lambda_3 & 0 \\ \nu_2 & 0 & 0 & 0 & -(\nu_2+\lambda_3) & \lambda_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \nu_3 & 0 & 0 & 0 & 0 & 0 & -\nu_3 \end{bmatrix} \qquad (6)$$

The second phase, corresponding to the point of inspection, is a discrete phase. At this point probabilities in the model are transferred between unrevealed condition states and known condition states. It is at this point that the decisions need to be made regarding maintenance, according to the values selected for M1, M2 and M3. At the inspection time t, probabilities are transferred according to equations 7 where $Q_i(t)$ and $Q_i'(t)$ are the state probabilities immediately prior to following inspection respectively

$$\begin{aligned} Q_1'(t) &= Q_1(t) \\ Q_2'(t) &= 0 \\ Q_3'(t) &= Q_3(t) + M1 \times Q_2(t) \\ Q_4'(t) &= Q_4(t) + (1-M1) \times Q_2(t) \\ Q_5'(t) &= 0 \\ Q_6'(t) &= Q_6(t) + M2 \times Q_5(t) \\ Q_7'(t) &= Q_7(t) + (1-M2) \times Q_5(t) \\ Q_8'(t) &= 0 \\ Q_9'(t) &= Q_9(t) + M3 \times Q_8(t) \end{aligned} \qquad (7)$$

D. Expected maintenance costs

Average repair costs for each type of maintenance work on each of the bridge elements of different materials were estimated from the database of previous work carried out. The average cost of the maintenance is combined with the possession costs (note that this cost is different depending on asset route criticality). The total repair cost over the structure life period is then calculated by taking the product of the number of bridge element repairs of each severity and the average costs of such repairs. The number of bridge element repairs can be calculated by integrating the rate of transitions from each corresponding degraded state (state 3, 6 and 9 in Figure 1) to new state over the predicted time, T. The expected repair costs are given in Equation 8. It is worth noting that not every maintenance action would require possession and that this is applied to the maintenance costs as appropriate. The servicing and inspection cost are also considered, depending on the frequency of the inspections and services, these costs can easily be added to the total costs. In total, the total expected maintenance costs for a component is:

Total expected maintenance cost
= Minor repair cost + Major repair cost + Replacement cost + Servicing cost + Inspection cost

$$= \int_0^T Q_3(t).v_1 dt \times C_1 + \int_0^T Q_6(t).v_2 dt \times C_2 + \int_0^T Q_9(t).v_3 dt \times C_3 + \sum_{f=1}^{T/f}\left[S_f\right] + \sum_{f=1}^{T/f}\left[I_f\right] \quad (8)$$

where

T = Length of the prediction period (year)
$Q_3(t)$ = Probability of the component requires minor repair at time t and has been scheduled to repair
$Q_6(t)$ = Probability of the component requires major repair at time t and has been scheduled to repair
$Q_9(t)$ = Probability of the component requires replacement at time t and has been scheduled to be replaced
v_1 = Minor repair rates of the component
v_2 = Major repair rates of the component
v_3 = Replacement rates of the component
C_1 = Average Minor Repair Costs
C_2 = Average Major Repair Costs
C_3 = Average Replacement Costs
S_f = Cost of servicing after every interval of f years
I_f = Cost of inspection after every interval of f years

V. Results

The following results demonstrate the capabilities of the model which has been produced for both the bridge components and the structure as a whole.

A. Maintenance Strategies

No repair (strategy 4)

Considering the metal decking, Figure 2 shows the probability of being in each possible condition state, over the 60 year life period, if there is no intervention during this time. The metal deck is assumed to be in the *good* condition (state 2) when the simulation started. Note that if no interventions are enabled in the model, the worst state (very poor state – state 8) becomes an absorbing state. At the end of 60 years, the probability of the deck being in a good condition is about 10% while the probability of being in poor and very poor condition is about 40% and 50% respectively.

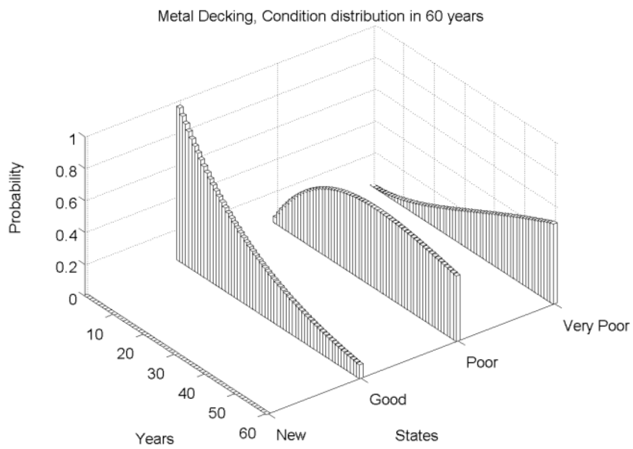

Fig 2. Probabilities of being in different states in the next 60 years of metal decks starting with good condition if there is no intervention.

Repair when the component condition reaches poor condition (strategy 2)

Consider a metal underbridge with the main components and their conditions as illustrated in figure 3. The maintenance strategy for the bridge was set to carry out repair when any of the bridge component reaches the poor condition.

Figure 4 shows the probability of the bridge being in the different condition states over the 60 year period. The bridge condition is, in this case, defined as the average condition of all the bridge components. Because the Cross girder (XG) and Bearings (BGL) are initially in the state where major repair is necessary, these components are immediately scheduled to be repaired. The effect of the repair process is reflected in the first 5 years when the probabilities of the bridge average condition being in a good and poor state decrease whilst the probability of being in a new condition increases. The model converges after about 15 years and it can be seen that following this repair strategy (repair when major repair is necessary), gives approximately a 50% probability that the average component condition will be in the ‘as new’ condition and the likelihood of the structure will require some form of minor repair and major repair is around 40% and 50% respectively. The probability of the component being in a very poor condition is negligible in this case.

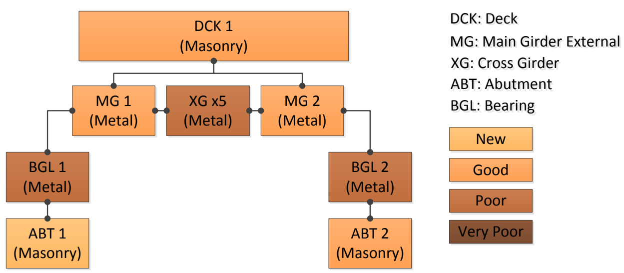

Fig 3. Structural arrangement of a metal underbridge.

The effects of the maintenance can be seen in the 'wave' nature of the plot. The peak of the 'wave' is when the inspection happens and the condition of the component is revealed. Following this point, any revealed failures are being scheduled for repair thus the probability of being in an 'as new' condition increases. A certain time after the repair, as the component continues to deteriorate, the probability of being in the 'as new' state decreases and the probabilities of being in poorer condition states increases. This process is what creates the 'wave' shape in the plot. After the next inspection when the component condition is revealed, the process is repeated again.

Figure 5 shows that the cost of each maintenance task on each bridge component. The initial cost is influenced by the work done on XG and BGL. These components being scheduled for repair at the beginning of the simulation is reflected in the plot where the cost of maintenance are at a peak during this period. The cumulative cost line shows the total expected repair cost after 60 years is around £200,000.

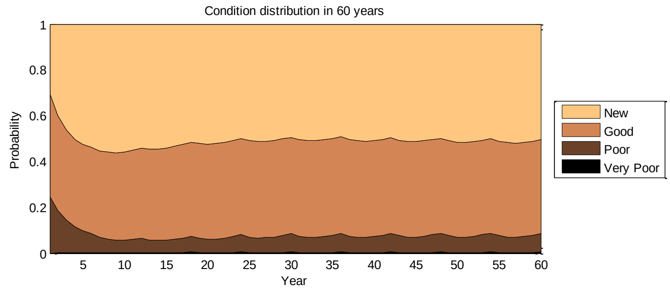

Fig 4. Probabilities of being in different conditions of a bridge

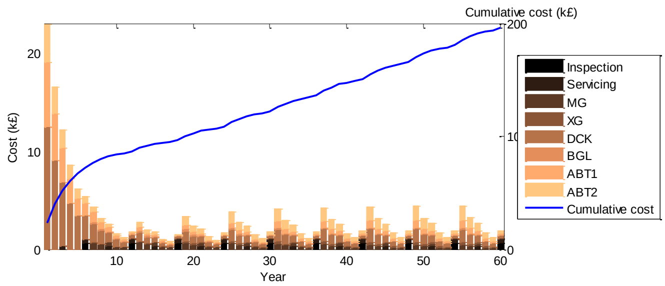

Fig 5. Expected maintenance cost for each components every year and cumulative maintenance cost for the whole bridge.

B. Scheduling of repair

With maintenance work on structures there is commonly a considerable delay time between when the decision is made to carry out the work and when it is started. This section investigates the effect which this has on the state of the structure. Figure 7 shows the probability of an asset being in a poor condition when different times for scheduling the maintenance are considered. If a major repair is being delayed from three months to a year due to scheduling, the average probability of being in poor state increases by about 3%. This is because as delay time increases, the repair rate decreases, which means the likelihood of a component being in poor condition is higher. Therefore delaying a repair will slightly degrade the condition of the asset, while the WLCC will be slightly cheaper. This presents the opportunity to explore a wider range of more flexible maintenance strategies where delaying repair can be used in tight budget situations or combined with opportunistic maintenance.

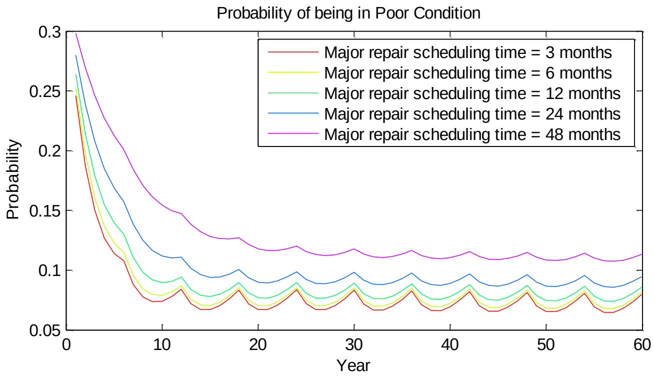

Fig 7. Delaying a major repair increases the probability of bridge being in in a poor condition.

VI. CONCLUSIONS

This paper demonstrates a Markov modelling approach to predict the condition of individual bridge elements along with the effects that interventions such as servicing, repair, and replacement will produce. For each bridge element the degradation process is determined by examining the maintenance records and analysing the times that each element takes to deteriorate to the point where a certain type of intervention is required. By combining the elemental models, a bridge structure model is formed what it can be used to investigate different maintenance strategies.

The model is capable of modelling the elements accounting for: current condition, material types, structure type, asset criticality, environment, inspection intervals, servicing intervals, repair strategy and the repair scheduling (delay) times. The model outputs are the probabilities of being in different states at any given time in the future; the expected maintenance cost for each type of intervention for each bridge component; and the total expected maintenance expenditure –WLCC over the entire prediction period.

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REFERENCES

1. Jiang, Y. and K.C. Sinha, *Bridge service life prediction model using the Markov chain.* Transportation Research Record, 1989(1223).
2. Robelin, C.-A. and S.M. Madanat, *History-Dependent Bridge Deck Mainenance and Replacement Optimization with Markov Decision Process.* Journal of Infrastructure Systems, 2007. **13**(3): p. 195-201.
3. Cesare, M.A., et al., *Modelling Bridge Deterioration With Markov Chains.* Journal of Transportation Engineering, 1991. **118**(6): p. 2237.
4. Ortiz-García, J.J., S.B. Costello, and M.S. Snaith3, *Derivation of Transition Probability Matrices for*

Pavement Deterioration Modeling. Journal of Transportation Engineering, 2006. **132**(2): p. 141-161.

5. Chase, S.B. and L. Gaspa, *Modelling the reduction in load capacity of Highway Bridges with age.* Journal of Bridge Engineering, 2000. **5**(4).
6. Morcous, G., *Performance Prediction of Bridge Deck Systems Using Markov Chains.* Journal of Performance of Constructed Facilities, 2006. **Vol. 20**(No. 2): p. 146–155.
7. S-K, N. and F. Moses, *Prediction of Bridge Service Life Using Time-dependent Reliability Analysis*, in *Bridge Management 3: Inspection, Maintenance, Assessment and Repair*1996, E&FNSpon.
8. Kleiner, Y., *Scheduling inspection and renewal of large infrastructure assets.* Journal of Infrastructure Systems, 2001. **7**(4): p. 136-143.
9. Sobanjo, J., P. Mtenga, and M. Rambo-Rodenberry, *Reliability-Based Modelling of Bridge Deterioration.* Journal of Bridge Engineering, 2010.
10. Mishalani, R.G. and S.M. Madanate, *Computational of infrastructure transition probabilities usnig stochastic duration models.* Journal of Infrastructure, 2002. **8**(4): p. 139-148.
11. Yang, Y.N., H.J. Pam, and M.M. Kumaraswamy, *Framework Development of Performance Prediction Models for Concrete Bridges.* Journal of Transportation Engineering, 2009. **135**(8): p. 545-554.
12. Agrawal, A.K., A. Kawaguchi, and Z. Chen, *Deterioration rates of typical bridge elements in New York.* Journal of Bridge Engineering, 2010. **15**: p. 419.
13. Frangopol, D.M., S.J. Kong, and S.E. Gharaibeh, *Reliability-based life-cycle management of Highway bridges.* Journal of Computing in Civil Engineering, 2001. **15**(1): p. 27-34.